Cross-Cross Resonance Gate

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The implementation of high-fidelity swapping operations is of vital importance for executing quantum algorithms on a quantum processor with limited connectivity. We present an efficient pulse-control technique, the cross-cross resonance (CCR) gate, to implement iSWAP and SWAP gates with dispersively coupled fixed-frequency transmon qubits. The key to the operation of the CCR gate is the simultaneous driving of both of the coupled qubits at the frequency of another qubit, whereby a fast two-qubit interaction roughly equivalent to $ZX + XZ$ entangling gates is realized without strongly driving the qubits. We develop a calibration technique for the CCR gate and evaluate the performance of the iSWAP and SWAP gates. The CCR gate shows remarkable improvements in the average gate error from the conventional decomposition based on the cross-resonance gate.

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I. INTRODUCTION

The number of qubits available in a superconducting quantum processor has been continuously growing despite the restricted connectivity exhibited by square and heavy-hexagon lattice structures [1]. The connections in these structures are carefully designed to avoid overlapping transition frequencies with neighboring qubits while enabling the implementation of an error-correction code. This is the key ingredient of fault-tolerant quantum processors [2–7]. Although such processors can implement state-of-the-art two-qubit entangling gates with an average gate error of less than 10^{-2} [8], the insertion of multiple swapping operations to entangle nonconnected qubit pairs becomes a visible obstacle to building large-scale quantum processors [9]. Therefore, further improvement of entangling gates is indispensable.

With superconducting qubits, entangling gates are generally implemented using electric dipole interactions between qubits, and can be roughly divided into two subclasses. One class of implementations is realized by dynamically modulating the device parameters to switch the coupling with the aid of tunable elements [10–13].

The additional control lines for the tunable elements may induce an extra degree of interaction with a noisy environment, and such an architecture tends to show shorter coherence times. Another class of implementations uses the manipulation of Hamiltonian terms by irradiating a pair of dispersively coupled qubits with microwave pulses [14–19]. Such architectures can be realized without tunable elements. Thus, qubits can be well protected from the source of decoherence at the cost of a longer gate time [20], impeded by a crosstalk error due to the static ZZ interaction [21,22].

The cross-resonance (CR) gate is a typical implementation of the latter subclass, without any tunable elements [14,20,23]. In the CR gate, a ZX interaction is induced by irradiating one of the two coupled qubits with microwave radiation that resonates with the other qubit. This simple architecture offers benefits for the scaling up of devices [14,20,23]. However, in the CR gate, there is a trade-off between fast gate execution and adherence to adiabatic conditions. For fast gate execution, we need to increase the microwave pulse intensities, which makes the adiabatic condition break, resulting in adverse effects such as drive-induced saturation and population leakage to higher-order levels [24,25].

In this paper, we propose a pulse sequence, called the cross-cross resonance (CCR) gate, which relaxes the above trade-off. The CCR gate is a natural extension of the CR gate, where a $ZX + XZ$ interaction is induced by simultaneously irradiating both of the two coupled qubits with microwaves that resonate with the other qubit. Although both qubits are driven simultaneously, the

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required microwave intensities in the CCR gate are weaker than those in CR gates. Thus, the CCR gate avoids the adverse effects derived from strong drives while preserving the gate execution time.

This paper is organized as follows. In Sec. II, we first show the advantage of CCR gates over CR gates by theoretical calculations. In Sec. III, we next demonstrate a CCR gate on the IBM Quantum cloud provider using Qiskit Pulse [26,27]. The calibration of the CCR gate is scrutinized, and we introduce a channel purification technique that enables the investigation of an approximate unitary representation of experimental gates. The calibrated CCR gate demonstrates remarkable improvements in the average gate fidelity, which is confirmed by interleaved randomized benchmarking [28–30].

II. PRINCIPLE

We describe a model of the CCR gate using the Hamiltonian of the standard circuit-quantum-electrodynamics setup. A system consisting of two qubits dispersively coupled to a bus resonator can be modeled using Duffing oscillators:

$$\begin{aligned} \mathcal{H} = & \sum_{i \in [0,1]} \left\{ \tilde{\omega}_i b_i^\dagger b_i + \frac{\alpha_i}{2} b_i^\dagger b_i^\dagger b_i b_i \right\} \\ & + g (b_0^\dagger + b_0) (b_1^\dagger + b_1) \\ & + \sum_{i \in [0,1]} \Omega_i \cos(\omega_{di} t) (b_i^\dagger + b_i), \end{aligned} \quad (1)$$

where b_i ($b_i^\dagger$) is the annihilation (creation) operator, and $\tilde{\omega}_i$, α_i , and g are the dressed eigenfrequency, the anharmonicity of the i th qubit, and the coupling between the two qubits, respectively. The last term describes the general drive Hamiltonian for each qubit, with a time-invariant drive Ω_i and drive frequency ω_{di} . Let $i \in [0, 1]$ denote the labels for the control and target qubits, respectively.

A. Cross-resonance gate

We start with the Hamiltonian of the CR gate, which is the predecessor of our proposal. The CR gate is a microwave-only two-qubit entangling gate that is commonly used for fixed-frequency dispersively coupled qubits [23]. In the CR gate, the control qubit is irradiated with a microwave tone at the frequency of the target qubit $\omega_{d0} = \tilde{\omega}_1 \sim \omega_1 - g^2/\Delta$, where $\Delta = \tilde{\omega}_0 - \tilde{\omega}_1$ is the detuning of the two qubits. This stimulus induces a controlled rotation of the target qubit, whose direction depends on the state of the control qubit.

For simplicity, we model transmons as ideal qubits, ignoring classical crosstalk [20]. The effective Hamiltonian of the CR gate for a qubit model under the rotating-wave approximation $\Omega_0/(\tilde{\omega}_0 + \tilde{\omega}_1) \ll 1$, the strong dispersive

condition $g/\Delta \ll 1$, and the weak-drive condition $\Omega/\Delta \ll 1$ is written as follows [31]:

$$\mathcal{H}_{\text{eff}}^{\text{CR}} \sim \frac{\text{sgn}(\Delta) g \Omega_0}{\sqrt{\Delta^2 + \Omega_0^2}} \frac{ZX}{2}, \quad (2)$$

where ZX denotes the tensor product of the standard single-qubit Pauli operators Z and X . Note that the above notation for the effective Hamiltonian $\mathcal{H}_{\text{eff}}$ is a representation in a rotating frame moved from the laboratory frame using the unitary operator

$$R = e^{-i(\Omega_0^2/2\Delta)t(ZI/2)}. \quad (3)$$

To clarify the difference between CR and CCR gates, we characterize the two-qubit gate implemented by each pulse sequence using the KAK decomposition [32]. Any two-qubit gate can be described as a 4×4 matrix in $\text{SU}(4)$, which can be decomposed as follows:

$$U = k_1 A k_2, \quad (4)$$

where k_1 and k_2 are 4×4 matrices in $\text{SU}(2) \otimes \text{SU}(2)$, and A is a 4×4 matrix in the maximal Abelian subgroup $\mathcal{A}$ of $\text{SU}(4)$. The matrix A is parameterized as follows:

$$A(c_1, c_2, c_3) = \exp \left(-i \sum_{i=1}^3 c_i \frac{A_i}{2} \right), \quad (5)$$

where $c_{1,2,3} \in [0, \pi/2]$ are the Cartan coefficients, and $A_{1,2,3}$ is any combination of mutually commutative two-qubit Pauli operators, such as $\{XX, YY, ZZ\}$ and $\{ZX, XZ, YY\}$. The controlled-NOT (CNOT), iSWAP, and SWAP gates, which are typical two-qubit gates, have Cartan coefficients $(\pi/2, 0, 0)$, $(\pi/2, \pi/2, 0)$, and $(\pi/2, \pi/2, \pi/2)$, respectively. These coefficients illustrate the overhead of three CNOT gates required for the decomposition of the SWAP gate. The CR gate has Cartan coefficients as follows:

$$c(t) = \left(\frac{\text{sgn}(\Delta) g \Omega_0}{\sqrt{\Delta^2 + \Omega_0^2}} t, 0, 0 \right), \quad (6)$$

which is locally equivalent to a controlled-rotation gate, namely the fundamentals of the CNOT gate. Note that without the qubit approximation, terms derived from higher-order levels are added to the effective Hamiltonian [31].

B. Cross-cross resonance gate

The CCR gate is a natural extension of the CR gate, where the control and target qubits are simultaneously irradiated with microwave tones at the frequency of the other qubit, and $\omega_{d0} = \tilde{\omega}_1 - \Omega_0^2/2\Delta$ and $\omega_{d1} = \tilde{\omega}_0 + \Omega_1^2/2\Delta$. In

contrast to the CR gate, the drive frequencies of the CCR gate are a function of the drive amplitudes Ω_0 and Ω_1 owing to the drive-induced Stark shift [33,34]. We again use the qubit model for simplicity, and find that the effective Hamiltonian of the CCR gate can be described as follows:

$$\mathcal{H}_{\text{eff}}^{\text{CCR}} \sim \frac{\text{sgn}(\Delta)g\Omega_0}{\sqrt{\Delta^2 + \Omega_0^2 + \frac{3}{2}\Omega_1^2}} \frac{ZX}{2} - \frac{\text{sgn}(\Delta)g\Omega_1}{\sqrt{\Delta^2 + \frac{3}{2}\Omega_0^2 + \Omega_1^2}} \frac{XZ}{2}. \quad (7)$$

Given $\Omega_1 = 0$, the effective Hamiltonian of the CCR gate matches that of the CR gate shown in Eq. (2). Details are shown in Appendix A. The CCR gate has the following Cartan coefficients:

$$c(t) = \left(\frac{+\text{sgn}(\Delta)g\Omega_0}{\sqrt{\Delta^2 + \Omega_0^2 + \frac{3}{2}\Omega_1^2}}t, \frac{-\text{sgn}(\Delta)g\Omega_1}{\sqrt{\Delta^2 + \frac{3}{2}\Omega_0^2 + \Omega_1^2}}t, 0 \right). \quad (8)$$

C. Effective Hamiltonian of CR and CCR gates with higher-order excited levels

Next, we discuss effects derived from higher-order excited levels. For simplicity, we deal only with the leading-order terms up to first order in g/Δ and Ω/Δ . The effective Hamiltonian of the CR gate in the qutrit system [31] is known to be as follows:

$$\mathcal{H}_{\text{eff}}^{\text{CR}} = \frac{g\Omega_0}{\Delta} \left(\frac{\alpha_0}{\alpha_0 + \Delta} \right) \frac{ZX}{2} + \frac{g\Omega_0}{\Delta} \left(\frac{\Delta}{\alpha_0 + \Delta} \right) \frac{IX}{2}, \quad (9)$$

where an IX term, called the quantum crosstalk, appears and can be canceled by applying a cancellation drive to the target qubit [20]. On the other hand, the effective Hamiltonian of a CCR gate with cancellation drives can be calculated as follows:

$$\mathcal{H}_{\text{eff}}^{\text{CCR}} = \frac{g\Omega_0}{\Delta} \left(\frac{\alpha_0}{\alpha_0 + \Delta} \right) \frac{ZX}{2} - \frac{g\Omega_1}{\Delta} \left(\frac{\alpha_1}{\alpha_1 - \Delta} \right) \frac{XZ}{2}, \quad (10)$$

which is a simple sum of the effective Hamiltonians of bidirectional CR gates with cancellation drives. In the case of the qutrit model, the anharmonicity of each qutrit appears in the coefficients of the ZX and XZ terms, indicating that we should irradiate the two qubits with different drive amplitudes.

D. Advantages of CCR gates over CR gates

In what follows, we compare CCR and CR gates in terms of drive-induced saturation and leakage from the computational basis.

1. Tolerance to drive-induced saturation

As one can see from Eqs. (2) and (7), the strength of the entanglement-generation term is a linear function of the drive strength under the weak-drive condition, but it is known to be saturated by third-order terms when the drive amplitude becomes comparable to the detuning between the qubits. This phenomenon is one of the adverse effects originating from the violation of the adiabatic condition, and we call it drive-induced saturation. In CCR gates, the total power of the microwave drive is split between the qubits, and they are driven far below the saturation point. In other words, we can further reduce the gate time of CCR gates within a limited power budget. The gate times required to compose the SWAP operation are estimated as

$$t_{\text{SWAP}}^{\text{CR}} = \frac{3\pi\sqrt{\Delta^2 + \Omega^2}}{2g\Omega}, \quad t_{\text{SWAP}}^{\text{CCR}} = \frac{3\pi\sqrt{\Delta^2 + \frac{5}{8}\Omega^2}}{2g\Omega}, \quad (11)$$

where we assume a CCR drive amplitude $\Omega_0 = \Omega_1 = \Omega/2$, which is equivalent to a CR drive amplitude Ω , for the sake of a fair comparison between the two entangling gates. Because $\Delta^2, \Omega^2 > 0$, we find that $t_{\text{SWAP}}^{\text{CR}} > t_{\text{SWAP}}^{\text{CCR}}$; hence, the CCR gate can always implement faster swapping operations at the same drive strength.

2. Tolerance to leakage

Leakage from the computational basis is also one of the typical error sources in CR gates. To execute a fast CR gate, we need to increase the CR drive amplitude, which often excites the control qubit to higher-order levels. Therefore, leakage is a factor that often limits the execution speed of a CR gate. Similarly to drive-induced saturation, leakage from the computational basis is also a phenomenon originating from higher-order terms concerning drive amplitudes, and it is easy to expect that CCR gates will have a higher tolerance to this leakage than CR gates. We calculate the optimal CCR drive amplitude ratio that maximizes the gate execution speed of iSWAP and SWAP gates while suppressing the leakage as follows:

$$\frac{\Omega_1}{\Omega_0} = \frac{\alpha_1(\alpha_1 - \Delta)}{\alpha_0(\alpha_0 + \Delta)}. \quad (12)$$

Based on this condition, we compare the maximum amount of leakage in CCR and CR gates in Appendix B. Provided that the anharmonicities of the two qubits are comparable and the gates are calibrated at the same execution time, we find that CCR gates induce leakage less than that in CR gates by a factor of 2.

III. EXPERIMENT

A. System

The experiments presented in this paper are executed on qubits $Q3$ and $Q4$ of the five-qubit system `ibmq_bogota`

at IBM Quantum. This processor consists of dispersively coupled fixed-frequency transmon qubits, and the control electronics are capable of generating arbitrary waveforms with a cycle time $dt = 0.222$ ns, afforded by a fast digital-to-analog converter. The device parameters at the time of the experiment are shown in Table I.

The coupling strength between the two qubits is found to be $g/2\pi = 1.40$ MHz. Because the CCR gate drives the cross-resonance from both directions, the drive Hamiltonian consists of multiple frequency components. To realize a high-fidelity CCR gate, all transition frequencies, including higher energy levels, must be well isolated [1]. The selected qubit pair satisfies this requirement. Thus, it is considered to be a suitable experimental platform for our demonstration.

First, we independently calibrate a CR pulse with $Q3$ as the control and $Q4$ as the target qubit, and vice versa. The CR pulse envelopes with which $Q3$ and $Q4$ are irradiated correspond to $R_{ZX}(\pi/4) = e^{-i(\pi/8)ZX}$ and $R_{XZ}(\pi/4) = e^{-i(\pi/8)XZ}$ gates, respectively. Both CR pulse envelopes are “Gaussian square” pulses, that is, square pulses with Gaussian-shaped rising and falling edges. In addition, both pulses have a total duration of $t_{\text{CR}} = 848 dt = 188.4$ ns. The square portion of each of the pulses has a duration of $592 dt$, and the Gaussian rising and falling edges last $128 dt$ and have a standard deviation of $64 dt$. In `ibmq_bogota`, a standard CR gate is calibrated with a rotary echo, which eliminates almost all unwanted dynamics by driving the target qubit with an intense microwave tone at the resonance frequency. However, this technique cannot be applied to the CCR gate, because the induced local Hamiltonian terms $IX + XI$ may suppress the generator terms $XZ + ZX$ of the gate. For this reason, we calibrate the CR gates with active cancellation, wherein a target qubit is driven by a weak resonance tone in the opposite phase to the unwanted local Hamiltonian dynamics represented by $IX + IY$ [20]. The CR gates are carefully tuned using the selective Pauli error-amplifying technique that we recently developed [35]; this is akin to the technique of Hamiltonian error-amplifying tomography (HEAT) [36]. We should keep in mind that unwanted dynamics represented by a nonlocal ZZ interaction may still be alive and contribute slightly to the dynamics of CCR gates. See Appendix E for a detailed analysis.

TABLE I. Parameters of qubits.

	$Q3$	$Q4$
$\omega/2\pi$	4.858 GHz	4.978 GHz
$\alpha/2\pi$	−324 MHz	−338 MHz
T_1	112.4 μs	115.5 μs
T_2	191.7 μs	167.6 μs

B. Calibration of drive frequencies

Next, we calibrate the detuning of each microwave-tone frequency to realize the CCR gate by fixing the tone amplitude at the optimal point of the underlying CR gates. As mentioned in Sec. B, the existence of higher-order levels changes the frequency shift owing to the off-resonance microwave drives. Therefore, we need to simultaneously calibrate two drive frequencies, ω_{d0} and ω_{d1} . In the following, we change the notation of the qubit from $i = 0$ (1) to 3 (4) to clarify the correspondence with physical qubits. Note that we apply an active cancellation tone for another CR drive along with the CR tone for the qubit, which usually differs by a frequency of approximately $\Delta/(2\pi)$.

Figure 1 shows the experimental [Fig. 1(a)] and numerical [Fig. 1(b)] excited-state populations of the qubits after the simultaneous excitation while the drive frequencies are swept, where the excited-state population refers to the probability of existence in states other than the ground state. In our numerical simulation, we treat a transmon as a three-level system and apply the parameters shown in Table I, which are obtained in preliminary experiments. The experimental result is in qualitatively good agreement with the simulation result. The optimal detuning frequencies are highlighted by a star symbol where the two CR transitions cross over. This point is also experimentally confirmed to be at $(\omega_{d4} - \tilde{\omega}_3)/2\pi = -18.0$ MHz, $(\omega_{d3} - \tilde{\omega}_4)/2\pi = 8.9$ MHz.

In addition to the CR dynamics, several unwanted transitions are also confirmed experimentally. The g - f transition of $Q4$ indicates the frequency of resonance with the nonlinear two-photon transition process $|gg\rangle \rightarrow |gf\rangle$ occurring at $\tilde{\omega}_4 + \alpha_4/2$, which is approximately −50 MHz from $\tilde{\omega}_3$. The g - e transition $|gg\rangle \rightarrow |ge\rangle$ of this qubit is found at $\tilde{\omega}_4 \sim \tilde{\omega}_3 + \Delta$. The g - e transition $|gg\rangle \rightarrow |eg\rangle$ of $Q3$ is observed at $\tilde{\omega}_3 \sim \tilde{\omega}_4 - \Delta$. We also find a two-photon blue-sideband transition in which $|gg\rangle \rightarrow |ee\rangle$ is driven at $\tilde{\omega}_3 + \tilde{\omega}_4/2$, and this appears as yellow trajectories [37]. The optimal driving point is sufficiently isolated from these transitions, which indicates that we can generate a stable CCR gate with $Q3$ and $Q4$. See Appendix C for details.

C. Calibration of gate time

As previously discussed, it is not efficient to control the time evolution of the CCR gate with the pulse amplitude because the calibrated frequency offset, which is induced by the ac Stark effect, is a function of the drive amplitude. Thus, we calibrate the target gate by scanning over the gate time to find the optimal value, at which we realize an entangling gate *almost* locally equivalent to a $\sqrt{\text{iSWAP}}$ gate, which is a perfect entangler [32]. Owing to the nonzero ZZ interaction, a single CCR gate does not generate an entangling gate that is locally equivalent an iSWAP-type gate.

FIG. 1. Heatmaps of qubit excited-state population measured after CCR gates with various frequency detunings from the predetermined drive frequencies based on the individual CR gate calibration. (a) Experimental results and (b) simulation results. The measured populations of $Q3$ and $Q4$ are overlaid in the same plot to highlight an optimal driving point. The excited-state populations of $Q3$ and $Q4$ correspond to red and green, respectively. The yellow region indicates frequencies at which both $Q3$ and $Q4$ are simultaneously excited. See the discussion in the main text. In each panel, the vertical (horizontal) axis represents the detuning from the predetermined CR frequency of $Q4$ ($Q3$), corresponding to the frequency of the target qubit $Q3$ ($Q4$). The labels in (b) indicate the origin of each transition.

However, as is described later in Sec. III D, this interaction can be nullified with multiple CCR gates composed of iSWAP or SWAP gates. See Appendix E for details.

The target CCR gate is found when the following conditions for the Cartan coefficients are satisfied:

$$\begin{aligned} c_1(t) + c_2(t) &= \frac{\pi}{2}, \\ c_3(t) &= 0. \end{aligned} \quad (13)$$

To this end, we need to measure the Cartan coefficients while sweeping the CCR gate time. The evolution of a quantum system can be investigated using quantum process tomography (QPT) with measurement-error mitigation [38–41], which extracts a superoperator representation of a quantum process by tomographic reconstruction [42]. Because the Cartan coefficients are defined only on the unitary-matrix representation, we extract a unitary matrix from a superoperator estimated from a QPT experiment with the aid of a channel purification protocol. This technique is detailed in Appendix D. Figure 2(b) shows the approximate Cartan coefficients of the entangling gates while the total duration of the CCR drive is swept from $256 dt$ to $1600 dt$ with fixed rising and falling edges. It is found that two Cartan coefficients increase at the same rate and one remains at zero, as predicted by our effective-Hamiltonian model shown in Eq. (11). This indicates that the CCR drive generates entangling gates in the XY interaction family, in which the iSWAP gate belongs to [43]. From Fig. 2(b), we find the optimal total duration of the CCR drive to be $976 dt = 216.9$ ns.

FIG. 2. (a) Desired condition for an entangling gate generated by CCR drive. (b) Approximate Cartan coefficients of entangling gates while the CCR drive duration is swept.

As shown in Fig. 2(a), the unitary matrix corresponding to the CCR drive with the optimal time can be decomposed as follows:

$$R_{\text{CCR}\pi/4} = (U_{A1} \otimes U_{A2}) R_{\text{CCR}^*\pi/4} (U_{B1} \otimes U_{B2}), \quad (14)$$

where $U_{A1,A2,B1,B2}$ represents a 2×2 matrix, and $\text{CCR}_{\pi/4}^*$ represents a 4×4 matrix in the maximum Abelian subgroup of $\text{SU}(4)$.

D. Two-qubit randomized benchmarking with CCR gates

Finally, we implement iSWAP and SWAP gates consisting of multiple CCR gates and compare the performance of these gates with the standard gate decomposition based on CR gates. This is supported by the IBM Quantum cloud provider. We have already obtained an expression for the local rotations $U_{A1,A2,B1,B2}$ by the KAK decomposition of $\text{CCR}_{\pi/4}$. Therefore, we can generate $\text{CCR}_{\pi/4}^*$ gates as follows:

$$R_{\text{CCR}_{\pi/4}^*} = (U_{A1}^\dagger \otimes U_{A2}^\dagger) R_{\text{CCR}_{\pi/4}} (U_{B1}^\dagger \otimes U_{B2}^\dagger). \quad (15)$$

As shown in Figs. 3(a) and 3(b), we can implement the iSWAP and SWAP gates with echo sequences as follows:

$$U_{\text{iSWAP}} = (R_{\text{CCR}_{\pi/4}^*} R_{XY} \otimes R_{XY})^2, \quad (16)$$

$$U_{\text{SWAP}} = (R_{\text{CCR}_{\pi/4}^*} R_{XYZ} \otimes R_{XYZ})^3, \quad (17)$$

where $R_{XY} = e^{i(\pi/2)(X+Y/\sqrt{2})}$ and $R_{XYZ} = e^{i(\pi/3)(X+Y+Z/\sqrt{3})}$ represent single-qubit gates. The echoes are designed to circumvent the gate errors caused by the ZZ interaction derived from higher-order levels. Details are shown in Appendix E. On the other hand, the iSWAP and SWAP gates are also decomposed into two and three two-pulse-echoed Controlled-X (TPCX) gates, respectively, which are echoed versions of the CR gate [2]. In our setup, the TPCX gate is calibrated at a gate time of 334.2 ns. On the other hand, the CCR gate is calibrated at 216.9 ns, which is restricted by the slow speed of the interaction from $Q3$ to $Q4$.

Because the iSWAP and SWAP gates belong to the Clifford group, the performance of the gates can be precisely measured with a two-qubit interleaved randomized benchmarking (IRB) experiment. Figures 3(c) and 3(d) show the experimental results from an IRB experiment. We create ten random circuits for each Clifford-sequence length and take 1024 samplings of measurements for each random circuit to obtain a data point. The curves of the population other than that in the ground state are fitted by an exponential decay with the same scale and offset parameters in each figure. The performance of these gates and the coherence limits calculated from the calibrated gate time and from coherence times measured for idling qubits are summarized in Table II. From the table, it is found that the average gate infidelities obtained from IRB do not reach the coherence limits. However, it should be noted that coherence times during gate execution are generally

FIG. 3. (a) Gate sequence to generate iSWAP gate with two CCR gates. (b) Gate sequence to generate SWAP gate with three CCR gates. Experimental results from two-qubit interleaved randomized benchmarking for (c) iSWAP and (d) SWAP gates with the gate decomposition based on CR gates, supported by the IBM Quantum cloud provider, and with the gate decomposition based on CCR gates.

different from premeasured values due to the microwave irradiation [44–46].

To extract the contribution of the unitary error from the total error of the iSWAP and SWAP gates generated with CCR gates, we carefully analyze the Pauli transfer matrices previously measured by QPT experiments, as shown in Fig. 4(a). From the product of the inverse Pauli transfer matrices of the ideal gate and those obtained from the QPT experiments, we obtain the Pauli transfer matrices of the gate error. By applying dynamical generator analysis [47] to them, the unitary-matrix representations of the gate

TABLE II. Comparison of performance of iSWAP and SWAP gates implemented with CR and CCR gates.

	iSWAP		SWAP	
	CR	CCR	CR	CCR
Gate time (ns)	775.1	647.1	1073.8	935.1
Error (10^{-2})	1.4(1)	0.8(2)	3.5(9)	2.0(6)
Coherence limit	0.62	0.51	0.85	0.74

FIG. 4. (a) Pauli transfer matrices of iSWAP (left) and SWAP (right) gates generated with CCR gates, evaluated by QPT experiments with measurement-error mitigation. (b) Generator intensities of rotation errors in iSWAP (left) and SWAP (right) gates generated with CCR gates, extracted via a channel purification protocol.

errors are reconstructed; the ratio of the unitary error to the total gate error is 32% (iSWAP) and 26% (SWAP).

In general, QPT experiments are sensitive to state preparation and measurement (SPAM) errors, and we employ the measurement-error mitigation technique for accurate analysis. The average gate infidelities [48] are evaluated as 0.78×10^{-2} (iSWAP) and 2.85×10^{-2} (SWAP), which are almost comparable with the results of the IRB experiments. This indicates that our estimation of the unitary error is reliable, and that there is room for further reduction of the gate errors. Figure 4(b) shows the intensity of the generator for the reconstructed unitary error.

IV. SUMMARY AND DISCUSSION

We propose and demonstrate a CCR gate, which is a two-qubit control scheme for dispersively coupled fixed-frequency transmon qubits. In the CCR gate, two qubits are simultaneously driven at the frequency of another qubit, and this operation induces both ZX and XZ terms in the effective Hamiltonian. For equivalent ZX and XZ interaction strengths, the CCR gate generates an entangling gate approximately locally equivalent to the iSWAP rotation. Since the required microwave drive amplitudes are weaker than that for the CR gate, the CCR gate can suppress negative effects caused by the breaking of adiabatic conditions, such as drive-induced saturation and leakage from the computational basis. Our theoretical calculations show that iSWAP and SWAP gates implemented with the

CCR gate have half the leakage of those implemented with the CR gate for the same gate time.

In CCR gates, the drive amplitudes and drive frequencies are correlated by the drive-induced ac Stark shift caused by the bidirectional CR drives. We propose and demonstrate an efficient calibration technique to realize high-fidelity CCR gates to cope with the drive-induced Stark shift, along with a precise decomposition of iSWAP and SWAP gates that nullifies the nonlocal error term appearing in the CCR-gate Hamiltonian. By using interleaved randomized benchmarking, we obtain a gate error of $0.8(2) \times 10^{-2}$ at a gate time of 647.1 ns for the iSWAP gate, and $2.0(6) \times 10^{-2}$ at a gate time of 935.1 ns for the SWAP gate, which is better than the values for a conventional implementation based on CR gates.

Multimodal control pulses [19,45,46,49,50] can realize entangling gates that can potentially outperform existing entangling-gate implementations. This paper also shows an important achievement in terms of a reduction in the gate time and in leakage from the computational basis, which will pave the way for further device scaling with restricted qubit-to-qubit connectivity.

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APPENDIX A: EFFECTIVE-HAMILTONIAN MODEL OF CROSS-CROSS RESONANCE FOR A QUBIT MODEL

In the qubit model, the anharmonicity is infinite, so the qubit subspace is perfectly isolated, and the Hamiltonian is given by

$$\mathcal{H} = \omega_0 \frac{ZI}{2} + \omega_1 \frac{IZ}{2} + gXX + \Omega_0 \cos(\omega_{d0}t) XI + \Omega_1 \cos(\omega_{d1}t) IX, \quad (\text{A1})$$

where, for simplicity, we assume a constant-amplitude drive Ω_i on the X quadrature of the i th qubit. Moving into the frame rotating at $\omega_p = (\omega_{d0} + \omega_{d1})/2$ via the unitary operator

$$R = e^{-i\omega_p t(ZI/2 + IZ/2)} \quad (\text{A2})$$

gives

$$\begin{aligned}\mathcal{H} = & (\omega_0 - \omega_p) \frac{ZI}{2} + (\omega_1 - \omega_p) \frac{IZ}{2} + g \left(\frac{XX}{2} + \frac{YY}{2} \right) \\ & + \Omega_0 \left(\cos(\omega_m t) \frac{XI}{2} - \sin(\omega_m t) \frac{YI}{2} \right) \\ & + \Omega_1 \left(\cos(\omega_m t) \frac{IX}{2} + \sin(\omega_m t) \frac{IY}{2} \right),\end{aligned}\quad (\text{A3})$$

where $\omega_m = (\omega_{d0} - \omega_{d1})/2$. The Schrieffer-Wolff transformation [51] via the skew-Hermitian operator

$$S = -i \frac{g}{\Delta} \left(\frac{XY}{2} - \frac{YX}{2} \right) \quad (\text{A4})$$

gives the block-diagonal Hamiltonian as follows:

$$\begin{aligned}\mathcal{H} \sim & (\tilde{\omega}_0 - \omega_p) \frac{ZI}{2} + (\tilde{\omega}_1 - \omega_p) \frac{IZ}{2} \\ & + \Omega_0 \left(\cos(\omega_m t) \frac{XI}{2} - \sin(\omega_m t) \frac{YI}{2} \right) \\ & + \Omega_1 \left(\cos(\omega_m t) \frac{IX}{2} + \sin(\omega_m t) \frac{IY}{2} \right) \\ & + \frac{g\Omega_0}{\Delta} \left(\cos(\omega_m t) \frac{ZX}{2} - \sin(\omega_m t) \frac{ZY}{2} \right) \\ & - \frac{g\Omega_1}{\Delta} \left(\cos(\omega_m t) \frac{XZ}{2} + \sin(\omega_m t) \frac{YZ}{2} \right),\end{aligned}\quad (\text{A5})$$

where $\tilde{\omega}_0 = \omega_0 + g^2/\Delta$ and $\tilde{\omega}_1 = \omega_1 - g^2/\Delta$. Note that we truncate terms included in $O((g/\Delta)^3)$. Next, to consider the drive-induced Stark shift, moving into the rotating frame via the unitary operator

$$R = e^{-i\omega_m t (ZI/2 - IZ/2)} \quad (\text{A6})$$

gives

$$\begin{aligned}\mathcal{H} \sim & (\tilde{\omega}_0 - \omega_{d0}) \frac{ZI}{2} + (\tilde{\omega}_1 - \omega_{d1}) \frac{IZ}{2} \\ & + \Omega_0 \frac{XI}{2} + \Omega_1 \frac{IX}{2} \\ & + \frac{g\Omega_0}{\Delta} \left(\cos(2\omega_m t) \frac{ZX}{2} - \sin(2\omega_m t) \frac{ZY}{2} \right) \\ & - \frac{g\Omega_1}{\Delta} \left(\cos(2\omega_m t) \frac{XZ}{2} + \sin(2\omega_m t) \frac{YZ}{2} \right).\end{aligned}\quad (\text{A7})$$

The Schrieffer-Wolff transformation via the skew-Hermitian operator

$$S = -i \left(\frac{\Omega_0}{\Delta_0} \frac{YI}{2} + \frac{\Omega_1}{\Delta_1} \frac{IY}{2} \right), \quad (\text{A8})$$

where $\Delta_i = \tilde{\omega}_i - \omega_{di}$, gives the block-diagonal Hamiltonian as follows:

$$\begin{aligned}\mathcal{H} \sim & \Delta_0 \left(1 + \frac{r_0^2}{2} \right) \frac{ZI}{2} + \Delta_1 \left(1 + \frac{r_1^2}{2} \right) \frac{IZ}{2} \\ & - \frac{\Omega_0 r_0^2}{3} \frac{XI}{2} - \frac{\Omega_1 r_1^2}{3} \frac{IX}{2} \\ & + \left\{ \frac{g\Omega_0}{\Delta} \left(1 - \frac{r_0^2 + r_1^2}{2} \right) + \frac{g\Omega_1}{\Delta} r_0 r_1 \right\} \cos(2\omega_m t) \frac{ZX}{2} \\ & - \left\{ \frac{g\Omega_1}{\Delta} \left(1 - \frac{r_0^2 + r_1^2}{2} \right) + \frac{g\Omega_0}{\Delta} r_0 r_1 \right\} \cos(2\omega_m t) \frac{XZ}{2} \\ & + \frac{g}{\Delta} (r_1 \Omega_1 - r_0 \Omega_0) \cos(2\omega_m t) \frac{XX}{2} \\ & + \frac{g}{\Delta} (r_1 \Omega_0 - r_0 \Omega_1) \cos(2\omega_m t) \frac{ZZ}{2} \\ & - \frac{g\Omega_0}{\Delta} \left(1 - \frac{r_0^2}{2} \right) \sin(2\omega_m t) \frac{ZY}{2} \\ & - \frac{g\Omega_1}{\Delta} \left(1 - \frac{r_1^2}{2} \right) \sin(2\omega_m t) \frac{YZ}{2} \\ & + \frac{g\Omega_0}{\Delta} r_0 \sin(2\omega_m t) \frac{XY}{2} \\ & + \frac{g\Omega_1}{\Delta} r_1 \sin(2\omega_m t) \frac{YX}{2},\end{aligned}\quad (\text{A9})$$

where $r_i = \Omega_i/\Delta_i$. Note that we truncate terms included in $O((\Omega_0/\Delta)^4)$ and $O((\Omega_1/\Delta)^4)$. We set the microwave drive frequencies ω_{di} to

$$\omega_{d0} = \omega_1 - \frac{\Omega_0^2}{2\Delta}, \quad (\text{A10})$$

$$\omega_{d1} = \omega_0 + \frac{\Omega_1^2}{2\Delta} \quad (\text{A11})$$

to satisfy the following conditions:

$$\Delta_0 \left(1 + \frac{r_0^2}{2} \right) = -\Delta_1 \left(1 + \frac{r_1^2}{2} \right) = -2\omega_m. \quad (\text{A12})$$

We can rewrite the Hamiltonian as follows:

$$\begin{aligned}\mathcal{H} \sim & \Delta \left\{ 1 + \frac{1}{2} \left(\frac{\Omega_0}{\Delta} \right)^2 \right\} \frac{ZI}{2} - \Delta \left\{ 1 + \frac{1}{2} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \frac{IZ}{2} \\ & - \frac{\Omega_0}{3} \left(\frac{\Omega_0}{\Delta} \right)^2 \frac{XI}{2} - \frac{\Omega_1}{3} \left(\frac{\Omega_1}{\Delta} \right)^2 \frac{IX}{2} \\ & + \frac{g\Omega_0}{\Delta} \left\{ 1 - \frac{1}{2} \left(\frac{\Omega_0}{\Delta} \right)^2 - \frac{3}{2} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \cos(2\omega_m t) \frac{ZX}{2}\end{aligned}$$

$$\begin{aligned}
& -\frac{g\Omega_1}{\Delta} \left\{ 1 - \frac{3}{2} \left(\frac{\Omega_0}{\Delta} \right)^2 - \frac{1}{2} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \cos(2\omega_m t) \frac{XZ}{2} \\
& + \left(\frac{\Omega_1}{\Delta} \right)^2 \cos(2\omega_m t) \sin(4\omega_m t) \left\{ \frac{YX}{2} \right. \\
& - g \left\{ \left(\frac{\Omega_0}{\Delta} \right)^2 + \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \cos(2\omega_m t) \frac{XX}{2} \\
& + g \left(\frac{\Omega_1}{\Delta} \right)^2 \sin(4\omega_m t) \sin(2\omega_m t) \frac{YY}{2}. \quad (A15) \\
& - 2g \frac{\Omega_0}{\Delta} \frac{\Omega_1}{\Delta} \cos(2\omega_m t) \frac{ZZ}{2} \\
& - \frac{g\Omega_0}{\Delta} \left\{ 1 - \frac{1}{2} \left(\frac{\Omega_0}{\Delta} \right)^2 \right\} \sin(2\omega_m t) \frac{ZY}{2} \\
& - \frac{g\Omega_1}{\Delta} \left\{ 1 - \frac{1}{2} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \sin(2\omega_m t) \frac{YZ}{2} \\
& + g \left(\frac{\Omega_0}{\Delta} \right)^2 \sin(2\omega_m t) \frac{XY}{2} \\
& + g \left(\frac{\Omega_1}{\Delta} \right)^2 \sin(2\omega_m t) \frac{YX}{2}. \quad (A13)
\end{aligned}$$

Finally, moving into the rotating frame via the unitary operator

$$R = e^{i2\omega_m t(ZI/2 - IZ/2)} \quad (A14)$$

gives

$$\begin{aligned}
\mathcal{H} \sim & -\frac{\Omega_0}{3} \left(\frac{\Omega_0}{\Delta} \right)^2 \left\{ \cos(2\omega_m t) \frac{XI}{2} - \sin(2\omega_m t) \frac{YI}{2} \right\} \\
& - \frac{\Omega_1}{3} \left(\frac{\Omega_1}{\Delta} \right)^2 \left\{ \cos(2\omega_m t) \frac{IX}{2} + \sin(2\omega_m t) \frac{IY}{2} \right\} \\
& + \frac{g\Omega_0}{\Delta} \left\{ 1 - \frac{1}{2} \left(\frac{\Omega_0}{\Delta} \right)^2 - \frac{3}{4} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \frac{ZX}{2} \\
& - \frac{3}{4} \frac{g\Omega_0}{\Delta} \left(\frac{\Omega_1}{\Delta} \right)^2 \left\{ \cos(4\omega_m t) \frac{ZX}{2} + \sin(4\omega_m t) \frac{ZY}{2} \right\} \\
& - \frac{g\Omega_1}{\Delta} \left\{ 1 - \frac{3}{4} \left(\frac{\Omega_0}{\Delta} \right)^2 - \frac{1}{2} \left(\frac{\Omega_1}{\Delta} \right)^2 \right\} \frac{XZ}{2} \\
& - \frac{3}{4} \frac{g\Omega_1}{\Delta} \left(\frac{\Omega_0}{\Delta} \right)^2 \left\{ \cos(4\omega_m t) \frac{XZ}{2} - \sin(4\omega_m t) \frac{YZ}{2} \right\} \\
& - 2g \frac{\Omega_0}{\Delta} \frac{\Omega_1}{\Delta} \cos(2\omega_m t) \frac{ZZ}{2} \\
& - g \left\{ \left(\frac{\Omega_0}{\Delta} \right)^2 + \left(\frac{\Omega_1}{\Delta} \right)^2 \cos(4\omega_m t) \right\} \cos(2\omega_m t) \frac{XX}{2} \\
& - g \left(\frac{\Omega_1}{\Delta} \right)^2 \cos(4\omega_m t) \sin(2\omega_m t) \frac{XY}{2} \\
& + g \left\{ \left(\frac{\Omega_0}{\Delta} \right)^2 \sin(2\omega_m t) \right.
\end{aligned}$$

The above Hamiltonian $\mathcal{H}$ can be divided into a stationary term and a perturbation with a period $T = \pi/\omega_m$. We can calculate the effective Hamiltonian from the Magnus expansion as follows:

$$\mathcal{H}_{\text{eff}} \sim \frac{\text{sgn}(\Delta)g\Omega_0}{\sqrt{\Delta^2 + \Omega_0^2 + \frac{3}{2}\Omega_1^2}} \frac{ZX}{2} - \frac{\text{sgn}(\Delta)g\Omega_1}{\sqrt{\Delta^2 + \frac{3}{2}\Omega_0^2 + \Omega_1^2}} \frac{XZ}{2}. \quad (A16)$$

Note that the above notation for the effective Hamiltonian $\mathcal{H}_{\text{eff}}$ is a representation of a rotating frame moved from the laboratory frame using the unitary operator

$$R = e^{-i(\Omega_0^2/2\Delta)t(ZI/2)} e^{i(\Omega_1^2/2\Delta)t(IZ/2)}. \quad (A17)$$

APPENDIX B: COMPARISON OF CCR AND CR GATES IN QUTRIT MODEL

From Eq. (7), we can calculate the Cartan coefficients during a CCR gate in a two-qutrit system under the weak-drive condition $O(\Omega/\Delta)$ as follows:

$$c(t) = \left(\frac{+g\Omega_0}{\Delta} \frac{\alpha_0}{(\alpha_0 + \Delta)} t, \frac{-g\Omega_1}{\Delta} \frac{\alpha_1}{(\alpha_1 - \Delta)} t, 0 \right). \quad (B1)$$

It can be seen that with the echo sequence shown in Figs. 3(a) and 3(b), the execution time of iSWAP and SWAP gates constructed with CCR gates is determined by the sum of the growth rates of the Cartan coefficients,

$$v_{\text{ent}} = \frac{g}{\Delta} \left\{ \left| \frac{\alpha_0 \Omega_0}{\alpha_0 + \Delta} \right| + \left| \frac{\alpha_1 \Omega_1}{\alpha_1 - \Delta} \right| \right\}. \quad (B2)$$

The application of the CCR drive to the qutrit system makes the adiabatic condition break and gives the maximum amount of leakage under the strong dispersive condition $O(g/\Delta)$ as follows:

$$\xi_{\text{max}} = \left(\frac{\Omega_0}{\alpha_0 + \Delta} \right)^2 + \left(\frac{\Omega_1}{\alpha_1 - \Delta} \right)^2. \quad (B3)$$

Note that we can also treat normal CR gates by setting $\Omega_0 = 0$ or $\Omega_1 = 0$. We assume that the best entangling gate minimizes the gate time while suppressing the maximum

amount of leakage, which can be expressed as follows:

$$\begin{aligned} & \text{maximize} \quad \nu_{\text{ent}} \\ & \text{subject to} \quad \xi_{\text{max}} = \xi. \end{aligned} \quad (\text{B4})$$

Here, the latter constraint implies that the CCR drive amplitudes Ω_0 and Ω_1 satisfy an elliptic relation as follows:

$$\begin{cases} \Omega_0 = \sqrt{\xi} (\alpha_0 + \Delta) \cos \theta, \\ \Omega_1 = \sqrt{\xi} (\alpha_1 - \Delta) \sin \theta, \end{cases} \quad (\text{B5})$$

where $\theta \in [0, 2\pi]$ denotes an angle. Then, Eq. (B4) can be rewritten as follows:

$$\text{maximize} \quad |\alpha_0 \cos \theta| + |\alpha_1 \sin \theta|. \quad (\text{B6})$$

One of the solutions is given as follows:

$$\theta^* = \arctan \left(\frac{\alpha_1}{\alpha_0} \right). \quad (\text{B7})$$

Finally, by substituting Eq. (B7) into Eq. (B5), we obtain the optimal ratio of CCR drive amplitudes as follows:

$$\frac{\Omega_1}{\Omega_0} = \frac{\alpha_1(\alpha_1 - \Delta)}{\alpha_0(\alpha_0 + \Delta)}. \quad (\text{B8})$$

Then, we can compare the gate execution speeds of the CR and CCR gates for a fixed maximum amount of leakage ξ as follows:

$$\nu_{\text{ent}}^{\text{CR}(Q0 \rightarrow Q1)} = \frac{g}{\Delta} \sqrt{\frac{\xi}{2}} \alpha_0, \quad (\text{B9})$$

$$\nu_{\text{ent}}^{\text{CR}(Q1 \rightarrow Q0)} = \frac{g}{\Delta} \sqrt{\frac{\xi}{2}} \alpha_1, \quad (\text{B10})$$

$$\nu_{\text{ent}}^{\text{CCR}} = \frac{g}{\Delta} \sqrt{\frac{\xi}{2}} \sqrt{\alpha_0^2 + \alpha_1^2}. \quad (\text{B11})$$

Similarly, the maximum amount of leakage for a fixed gate execution speed ν is written as follows:

$$\xi_{\text{max}}^{\text{CR}(Q0 \rightarrow Q1)} = \frac{2}{\alpha_0^2} \left(\frac{\Delta \nu}{g} \right)^2, \quad (\text{B12})$$

$$\xi_{\text{max}}^{\text{CR}(Q1 \rightarrow Q0)} = \frac{2}{\alpha_1^2} \left(\frac{\Delta \nu}{g} \right)^2, \quad (\text{B13})$$

$$\xi_{\text{max}}^{\text{CCR}} = \frac{2}{\alpha_0^2 + \alpha_1^2} \left(\frac{\Delta \nu}{g} \right)^2. \quad (\text{B14})$$

From Eqs. (B11) and (B14), we can conclude that the CCR gate can achieve a speed $\sqrt{2}$ times faster or 2 times less leakage than the CR gate when the anharmonicity of the qubits is equal.

APPENDIX C: CCR DRIVE FREQUENCIES WITH VARIOUS DRIVE AMPLITUDES

As the CCR gate is driven, the eigenfrequencies of both qubits are shifted by the drive-induced Stark shift. Here, we observe the dependence of the operating point of the CCR drive frequency on the CCR drive strength. In this experiment, we first independently calibrate the $R_{ZX}(\pi/4)$ and $R_{XZ}(\pi/4)$ gates using CR gates with gate times of 145.8, 168.0, and 190.2 ns. Note that the pulse amplitude and gate time are inversely proportional. Next, while maintaining the respective drive amplitudes, CR drives from both directions are applied simultaneously at various drive frequencies. Figure 5 illustrates that the eigenfrequencies of $Q3$ and $Q4$ are functions of the gate time, which are

FIG. 5. Heatmaps of qubit excited-state populations measured after CCR gates with various frequency detunings from predetermined drive frequencies based on individual CR gate calibration. The measured populations of $Q3$ and $Q4$ are overlaid in the same plot. In each part, the vertical (horizontal) axis represents the detuning from the predetermined CR frequency of $Q4$ ($Q3$), corresponding to the frequency of the target qubit $Q3$ ($Q4$). The total duration of the $R_{ZX}(\pi/4)$ gate is set to (a) 145.8, (b) 168.0, and (c) 190.2 ns.

FIG. 6. Pauli transfer matrices of entangling gates generated by a CCR drive with a duration of 216.9 ns before (left) and after (right) channel purification, which roughly corresponds to the point shown by a star in Fig. 2(b).

redshifted and blueshifted, respectively, with respect to the drive amplitude.

In Fig. 5(a), it can be seen that the cross-resonance from $Q4$ to $Q3$ and the g - f transition of $Q4$ overlap. This suggests that the irradiation of $Q3$ with the CR drive causes population leakage from the first excited state of $Q3$ to the second excited state of $Q4$. This phenomenon is not peculiar to CCR gates but also occurs in ordinary CR gates. We find that the two-dimensional drive-frequency sweep experiment provides important information to cope with these leakage errors.

APPENDIX D: CHANNEL PURIFICATION

To analyze the approximate Cartan coefficients of the experimentally generated quantum gate, we introduce a channel purification method that is derived from the McWeeny purification method [52]. In the channel purification method, the representation of a quantum channel $\mathcal{E}$ is first translated into a Choi representation according to the channel-state duality as follows:

$$\rho_{\text{Choi}} = \sum_{ij} \chi_{ij} |P_i\rangle \langle P_j|, \quad (\text{D1})$$

where $|P_i\rangle$ is the vectorized i th Pauli operator, and χ_{ij} is a coefficient of the corresponding Choi matrix of the channel. Here, the Chi matrix is defined as follows:

$$\mathcal{E}(\rho) = \sum_{ij} \chi_{ij} P_i \rho P_j. \quad (\text{D2})$$

Next, we project the Choi matrix as follows:

$$\rho_{\text{Choi}} \rightarrow |\psi_{\text{max}}\rangle \langle \psi_{\text{max}}|, \quad (\text{D3})$$

where $|\psi_{\text{max}}\rangle$ is the eigenvector of ρ_{choi} with the maximum eigenvalue. The projected Choi matrix is translated back to the Kraus representation according to the channel-state duality. When the Choi matrix is projected to rank 1, there is only one Kraus operator K , and such a quantum channel satisfies the condition of complete positivity but is not trace-preserving. To derive a unitary operator that approximates the Kraus operator, we project the Kraus operator as follows:

$$K = M_L \text{diag} \{r_i e^{i\theta_i}\} M_R \quad (\text{D4})$$

$$\rightarrow M_L \text{diag} \{e^{i\theta_i}\} M_L^{-1}, \quad (\text{D5})$$

where $M_{L,R}$ is a regular matrix, and $r_i e^{i\theta_i}$ is a complex number. Figure 6 shows the Pauli transfer matrices before and after the channel purification. From Fig. 6, it is found that the Pauli transfer matrix does not change significantly as a result of the channel purification, which suggests that the target quantum channel is originally close to a unitary channel. The average gate infidelity between the Pauli transfer matrices before and after the channel purification is 0.015, which includes residual SPAM errors that cannot be mitigated by measurement-error mitigation, and incoherent errors originating from decoherence while the CCR drive is applied.

Dynamical generator analysis [47], which is a similar method for reconstructing a unitary-matrix representation from a quantum channel, is not suitable for this study because it can be used only when the target quantum channel is included in branch 0.

FIG. 7. Cartan coefficients of unitary gate generated by CCR drive with a static ZZ error γZZ ($\gamma \in [0, 0.4]$), while the ratio between the bidirectional CR drive amplitudes $(1-x) : (1+x)$ is swept ($x \in [-1, 1]$). Each line color corresponds to a Cartan coefficient; namely, there are three lines, for c_1 , c_2 , and c_3 .

FIG. 8. Pulse schedules for iSWAP gates with different implementations: (a) CCR gates and (b) TPCX gates. Each horizontal line corresponds to a pulse channel associated with a qubit in a specific frame. Channel names are shown on the left side, with the frequency assigned to the channel. Curly arrows indicate the frequency shift of the channel. These frequency shifts are introduced to correct drive-induced Stark shifts. The pulse amplitudes of the real and imaginary parts of the CCR and CR gates are also shown by triangle signs, where the ratio of the CR pulse amplitudes with which channels $U6$ and $U7$ are irradiated is designed to compensate for the different execution rates of the CR gates owing to finite anharmonicities.

APPENDIX E: ANTICROSSING OF CARTAN COEFFICIENTS

As mentioned in Sec. C, iSWAP gates cannot be implemented with a single CCR gate, although Eq. (11) shows two nonzero Cartan coefficients. Here, we qualitatively estimate the imperfection of the CCR gate using a simplified Hamiltonian model. As discussed in Sec. B, the effective Hamiltonian of the CCR gate consists of ZX and XZ terms arising from the bidirectional cross-resonance drive. In addition, it is thought that IX , IY , XI , and YI terms arise from classical crosstalk, and a ZZ term arises when higher-order levels are considered. In principle, the terms derived from classical crosstalk can be eliminated by

adding active cancellation drives. Therefore, we model the effective Hamiltonian as follows:

$$H = \left(\frac{1-x}{2}\right) \frac{XZ}{2} + \left(\frac{1+x}{2}\right) \frac{ZX}{2} + \gamma \frac{ZZ}{2}, \quad (\text{E1})$$

where x is a parameter corresponding to the ratio between the bidirectional CR drive amplitudes, and γ is a perturbation coefficient for the static ZZ term. We numerically calculate the Cartan coefficients for the unitary gate $U = e^{i\pi/2H}$ with various values of x and γ . Figure 7 shows the results of the numerical simulation, where we calculate within $x \in [-1, 1]$ and $\gamma \in [0, 0.4]$. Recalling that an iSWAP-type gate can be represented by Cartan

```

1: for  $\omega_{d3} - \omega_4$  from  $-100$  MHz to  $+100$  MHz do
2:   for  $\omega_{d4} - \omega_3$  from  $-100$  MHz to  $+100$  MHz do
3:     Prepare an initial state  $|\psi_{\text{in}}\rangle = |00\rangle$ 
4:     Run CCR gate and get a final state  $|\psi_{\text{out}}\rangle = U_{\text{CCR}}(\omega_{d3}, \omega_{d4}, t_{\text{CR}}) |\psi_{\text{in}}\rangle$ 
5:     Measure  $M(\omega_{d3}, \omega_{d4}) = \langle \psi_{\text{out}} | ZI | \psi_{\text{out}} \rangle + \langle \psi_{\text{out}} | IZ | \psi_{\text{out}} \rangle$ 
6:   end for
7: end for
8: Find  $\omega_{d3}^*, \omega_{d4}^*$  minimizing  $M(\omega_{d3}, \omega_{d4})$  around crossing-point of the bidirectional CR transitions

```

Algorithm 1. Calibration of CCR drive frequencies

```

1: for  $t$  from  $256$  dt to  $1600$  dt do
2:   Run QPT with CCR gate  $U_{\text{CCR}}(\omega_{d3}^*, \omega_{d4}^*, t)$  and get complete positive and trace preserving map  $\mathcal{S}_{\text{CCR}}(\omega_{d3}^*, \omega_{d4}^*, t)$ 
3:   Apply channel purification on  $\mathcal{S}_{\text{CCR}}(\omega_{d3}^*, \omega_{d4}^*, t)$  and get Unitary matrix representation  $U_{\text{CCR}}(\omega_{d3}^*, \omega_{d4}^*, t)$ 
4:   Apply KAK decomposition on  $U_{\text{CCR}}(\omega_{d3}^*, \omega_{d4}^*, t)$  and get Cartan coefficients  $c_1(t) \geq c_2(t) \geq c_3(t)$ 
5: end for
6: Find  $t^*$  s.t.  $c_1(t^*) + c_2(t^*) = \pi/4$  from Linear fitting

```

Algorithm 2. Calibration of CCR drive duration

coefficients $c_1 = c_2$ and $c_3 = 0$, the nonzero ZZ interaction, which is a dominant error source without a rotary echo [33], induces an anticrossing of two Cartan coefficients, yielding $c_1 \neq c_2$. This simulation indicates that a high-fidelity iSWAP gate cannot be implemented with a single CCR gate with qubits with a finite ZZ error. On the other hand, suppression of the ZZ interaction [22,45,46] enables us to implement an iSWAP gate with a single CCR gate and significantly shortens the gate time.

APPENDIX F: PULSE SEQUENCES FOR iSWAP GATE

Figure 8 shows a comparison of calibrated pulse schedules for iSWAP gates with different implementations. The key component of the CCR gate is the bidirectional CR drive on control channel U_6 (U_7), which applies microwave tones to Q_3 (Q_4) in the frame of Q_4 (Q_3). Weak active-cancellation tones are simultaneously applied to the drive channels D_3 and D_4 , which apply microwave tones in the frame of the associated qubits. Local operations are realized by two consecutive $\sqrt{X}$ pulses and three virtual- Z gates on D_3 and D_4 prepended and appended to the CCR gates. On the other hand, the standard iSWAP decomposition generates a pulse sequence based on echoed CR gates, yielding a slightly longer schedule. In this decomposition, CR gates can be implemented with a rotary tone [53], which appears in D_3 and D_4 in parallel with CR tones with a comparable amplitude to those in $\sqrt{X}$ gates.

APPENDIX G: PSEUDOCODES FOR CALIBRATION PROCEDURE OF THE CCR GATE

To clarify the calibration procedure of a CCR gate, the relevant pseudocodes are summarized in Algorithms 1 and 2.

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