Forces on a Small Grain in the Nonlinear Plasma Wake of Another

I. H. Hutchinson

Forces on a small grain in the nonlinear plasma wake of another

I H Hutchinson*

*Plasma Science and Fusion Center and Department of Nuclear Science and Engineering,
Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

The transverse force on a spherical charged grain lying in the plasma wake of another grain is analysed to assess the importance of ion-drag perturbation, in addition to the wake-potential-gradient. The ion-drag perturbation is intrinsically one order smaller than the wake-potential force in the ratio of grain size (r_p) to Debye length (λ_{De}). So ion-drag perturbation is important only in nonlinear wakes. Rigorous particle-in-cell calculations of the force are performed in the nonlinear regime with two interacting grains. It is found that even for quite large grains, $r_p/\lambda_{De} = 0.1$, the force is dominated by the wake-potential-gradient. The wake potential structure can then help explain the preferred alignment of floating dust grains.

INTRODUCTION

A solid grain in the flowing-plasma wake of another, the 'upstream-grain', is known experimentally to experience forces that are substantially different from those in the unperturbed plasma flow. In addition to the mutually repulsive electric inter-grain force arising from both of them being negatively charged, the wake gives rise to additional electric fields. These wake-field can act in transverse directions, and cause either alignment[1, 2] or misalignment[3–6] of the two grains in the flow. The linear theoretical potential structure of the upstream grain has been used[7] to explain some of these observations. However, the ion drag forces on the grains, arising from the flow of ions, are also an important part of their force balance; and it has been claimed[8] that the perturbation to the ion drag force is sometimes more important than the potential structure of the wake, in determining the equilibrium and dynamics of the wake-grain. Nonlinear wake force, in addition to being a previously unsolved fundamental plasma problem, is key to dusty plasma physics and important in space plasmas.

Here are presented the first rigorous calculations of the transverse force on a small grain in the wake of another in the nonlinear regime. First it is argued analytically that ion-drag-force perturbation is significant only in the nonlinear regime, where the classic linearized dielectric response calculation of the wake fails. Particle simulation is then used to compare the actual total force experienced by the wake-grain with the electric field force that would arise purely from the nonlinear wake potential structure of the upstream-grain by itself.

LINEAR AND NONLINEAR WAKE FORCES

The nonlinearity of the wake of a floating upstream-grain depends on the ratio of its radius r_p (assumed spherical) to the electron Debye length λ_{De}. The floating potential is generally $\phi_p \sim 2 - 4$ times $-T_e/e$ (T_e is the electron temperature), so that the charge on the grain, $Q \approx 4\pi\epsilon_0 r_p \phi_p$ (when $r_p/\lambda_{De} \ll 1$) is proportional to its radius. The grain's normalized charge, $\bar{Q} \equiv Q/(4\pi\epsilon_0\lambda_{De}T_e/e) \approx (e\phi_p/T_e)(r_p/\lambda_{De})$, is a proxy for the normalized size of the grain. Taking singly charged ions, unperturbed electron and ion densities are equal: $n_e = n_i$. The external ion drift velocity past the grains will be expressed as a Mach number $M = v_d/c_s$ normalized to the (cold-ion) sound speed $c_s = \sqrt{T_e/m_i}$. We presume $M \sim 1$, as is the case in experiments near a plasma sheath. The ion temperature is supposed $T_i \ll T_e$. Using the standard Coulomb cut-off integration, ignoring the thermal ion velocity, the ion drag (F_d) on a small grain in normalized units is

$$\frac{F_d}{T_e n_e \lambda_{De}^2} \approx \left(\frac{\bar{Q}^2}{M^2}\right) 4\pi \ln \Lambda. \tag{1}$$

In the regime of interest $\Lambda \sim 1 + M^2/|\bar{Q}|$ [9] is not a very large quantity; so $\ln \Lambda$ is at most 'a few', unlike the classical situation for elementary charges, in which Λ is very large. The charge, Q, on even a small dust grain can be thousands of times e. [Note that by definition $n_e T_e \lambda_{De}^2 = \epsilon_0 (T_e/e)^2$. Justification of the precise $\ln \Lambda$ value in eq. (1) requires detailed discussion [10].]

The upstream grain's wake potential structure (see e.g. Fig. 1) has magnitude proportional to charge Q in the linear regime. In fact the peak wake potential (for $M \sim 1$) is [11] $\phi_{\max} \approx 2Q/(4\pi\epsilon_0\lambda_{De}) \approx 2\phi_p r_p/\lambda_{De}$.

The wake scale length in the radial direction, transverse to the background flow, is approximately λ_{De}, so the (maximum) transverse force ($F_{w\phi}$) that would arise from the wake potential gradient acting on a grain of the same charge can be expressed in dimensionless form as

$$\frac{F_{w\phi}}{T_e n_e \lambda_{De}^2} = -Q\frac{\partial\phi}{\partial r}\frac{1}{(T_e n_e \lambda_{De}^2)} \sim \bar{Q}^2 8\pi. \tag{2}$$

Consequently, the ratio of the transverse wake-field-force on a wake-grain to the total drag force on the upstream-grain is

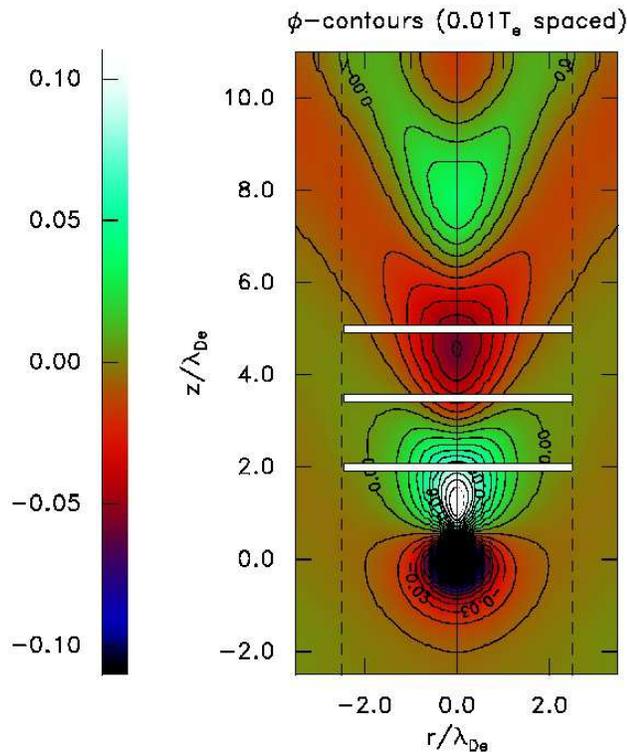$$F_{w\phi}/F_d \sim 2M^2/\ln \Lambda, \tag{3}$$

which is of order unity.

FIG. 1. Contours of the distribution of potential in units of T_e/e, in the wake of a point grain of normalized charge $\bar{Q} = 0.1$. The background flow is $M = 1$ in the +z-direction.

The transverse wake field and the drag force are the same order of magnitude because the drag force can be considered to arise (mostly) from the force exerted upon the upstream-grain by the ions focussed in its wake. The drag force thus arises physically from the *same* source (the ion-focus charge) as the transverse wake-field force, and is evaluated at roughly the same distance (for $M \sim 1$) from its source.

The *total* ion drag force on a wake grain is approximately the same as on the upstream-grain when they have equal charges. The transverse component of this ion drag force thus rivals the transverse wake potential force only if the *direction* of the wake drag force is at a large angle to the original flow direction.

Of course, for a free grain in the unperturbed plasma, the longitudinal force-balance *parallel* to the background flow requires ion drag to balance the background electric field, gravity and other forces if present. This balance is what determines the height at which a dust grain floats in a plasma sheath, for example. Then the *change* in the parallel drag force, arising from wake perturbation, is still intrinsically a factor (r_p/λ_{De}) smaller than the equilibrium value; whereas the wake potential force is of the same order as the drag force. [The enhancement of the ion density in the wake focus can be very large on axis[11], even when the wake potential is modest. So even though by ordering the change in parallel wake ion drag

is smaller than the wake potential force, drag changes are still significant on axis in the focus region.]

In summary, for *transverse* force balance or changes to *parallel* force on the wake-grain, the ion drag component is important only in the nonlinear wake regime, i.e. only to the extent that (r_p/λ_{De}) is not extremely small.

NUMERICAL SIMULATIONS

Direct numerical calculation of the dynamics of a wake in a collisionless plasma requires the full non-thermal ion distribution function to be tracked. Fluid-ion approximations are inadequate. It lends itself to Particle in Cell (PIC) simulation. The results here are obtained with the fully 3-dimensional "Cartesian-mesh, Oblique-boundary Particles and Thermals In Cell" (COPTIC) code, described in [11]. Electron density is presumed to be governed by Boltzmann distribution $n_e = n_{e\infty} \exp(e\phi/T_e)$ and ions are represented by particles accelerated by the self-consistent electric field and advanced in time and space by a standard leap-frog scheme, until a steady state is reached. This typically involves moving 30-200 million ions for about 1500 time-steps. The calculations here are collisionless.

The code is run with two grains of the same charge, the second located at some fixed position in the wake of the first. To compare the wake potential-gradient force with the total force, the wake potential from a single upstream grain is also calculated, as illustrated in Fig. 1. The regions in which the transverse wake force are later explored by introducing the second charge are marked with white bands.

The force on the grains is calculated in the manner described previously [10]. The Maxwell stress tensor and the electron pressure are integrated over a spherical surface, surrounding the grain whose force is to be obtained. Also the momentum flux of the ions crossing that sphere (both inward and outward) is accumulated and time-averaged in steady state. The total force is the sum of the three components: Maxwell stress, electron pressure, ion momentum flux. We perform this calculation for different (nested) spheres and observe that the totals (though not the individual components) are the same (when the simulation is converged).

Other PIC simulations of multiple grains have recently been published. Ikkurthi et al[12] report the transverse force only when the the grains are adjacent to one another, not in each other's wake. [They appear also to omit the electron pressure force.] Miloch et al[13] treat a grain truly in the wake of another but obtain the field force from a heuristic integration of inter-particle forces over a somewhat ill-defined region. Their results show some of the qualitative features of the present calculations, but do not appear to be quantitatively rigorous. Both of these other works address changes in the float-

ing potential of the wake-grain arising by ion focusing caused by the upstream grain. The present work, rather than requiring the grains to float, simply prescribes their potential (or equivalently charge).

The calculations use uniform external drifting-Maxwellian ion distributions with $T_i/T_e = 0.01$. COP-TIC can accommodate finite-sized objects (that absorb ions) or fixed point charges treated by a PPPM technique[14] to retain orbital accuracy, for which there is therefore no "direct ion collection" flux. Fig. 2 shows
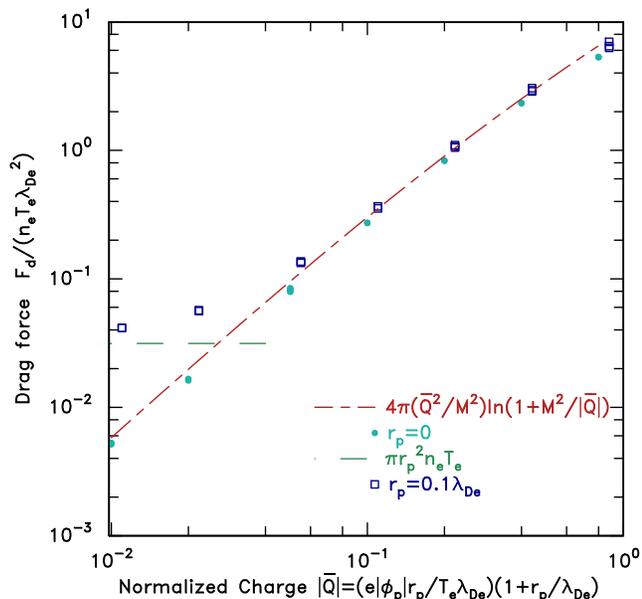

FIG. 2. Longitudinal force on a single isolated grain as a function of charge magnitude obtained by the COPTIC code. Two sets of simulations are shown, both with drift velocity $M = 1$. One has finite grain radius $r_p = 0.1\lambda_{De}$ (squares) and one has point charges (points, $r_p = 0$). The normalized charge is taken as $|\bar{Q}| = |\phi_p| r_p (1 + r_p/\lambda_{De})$.

calculations of the longitudinal (z-direction) drag force for isolated single grains with a range of charge and both finite-radius and infinitesimal radius. For each case, three points are actually plotted, corresponding to the force measurement at different spheres of radius between 0.2 and 0.5 times λ_{De}. These points coincide on the plot to within the size of the marker, giving an indication of how well the simulation is converged, and uncertainty from measurement noise (a few percent).

At large charge, the finite size of the grain is relatively unimportant. A force depending only on charge is obtained. This is consistent with the studies[11] which show that there is little difference in the wake fields between a point charge and a spherical object of finite radius even as large as $\lambda_{De}/10$. As the potential on the finite-radius grain drops below T_e/e, however, a saturation of the force is observed at a level approximately equal to the unperturbed momentum flux to the grain ($\pi r_p^2 n_e T_e$ at this velocity). For point grains this saturation does not occur,

the force is found to be about 10% below the value of eq. (1), which is within the theoretical uncertainty in that expression. Simulation systematic uncertainty may also be up to 10%.
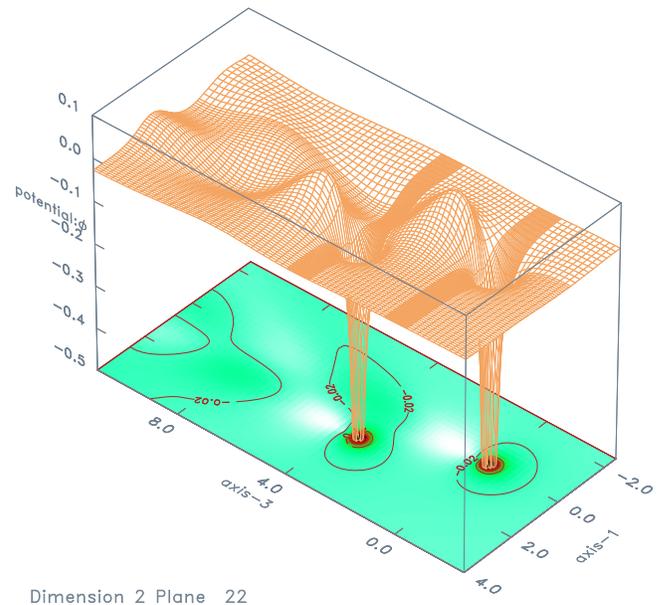

FIG. 3. Potential of two point charges of normalized value $Q/(4\pi\epsilon_0\lambda_{De}T_e/e) = -0.1$, in a flow $M = 1$ in the positive z-direction (axis-3). A two-dimensional slice through the 3-D domain at $y = 0$ is contoured and rendered. Charges are at positions $(0,0,0)$ and $(1.5,0,3.5)$ in units of λ_{De}. The deep negative potential wells around each charge are truncated only for visibility. Potential is in normalized units of T_e/e.

The potential obtained in a simulation with two charges is illustrated in Fig. 3. Non-uniform mesh spacing is used so as to obtain good resolution of the potential in the important regions without excessive mesh size ($64 \times 42 \times 100$ over a total domain of $6.5 \times 5 \times 13.5\lambda_{De}$). Both grains must be a distance of at least $2\lambda_{De}$ away from the mesh boundary to prevent spurious image forces caused by the potential boundary condition. In this case the wake grain is far enough downstream from the leading grain that it is adjacent to an axial potential valley, rather than the potential peak that immediately trails the leading grain.

Fig. 4 shows results, from a large number of runs like Fig. 3, for the transverse (x-direction) force on a point charge of normalized magnitude $Q/(4\pi\epsilon_0\lambda_{De}T_e/e) = -0.1$. This corresponds to the central point in Fig. 2 whose total (z) drag force is $0.26 n_e T_e \lambda_{De}^2$. The individual points are obtained directly from code runs with two charges of equal magnitude. One charge is at the origin and is the cause of the wake. The second, whose transverse force is measured by integration over a sphere surrounding it, is placed in the wake of the upstream charge, at different distances z downstream, plotted against its
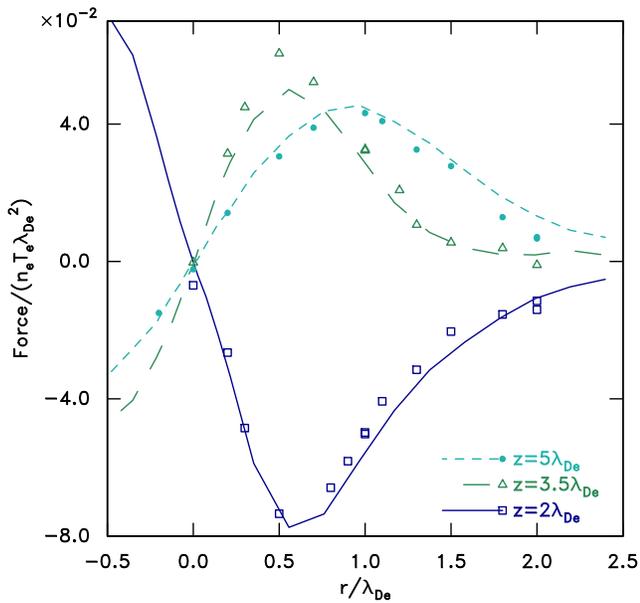
FIG. 4. Transverse force on a charge of normalized value -0.1 in the wake of an equal charge. The downstream distance is $z = 2$, 3.5, $5\lambda_{De}$ and the transverse position is r. Points: 2-grain direct simulation. Lines: pure electric wake-field-force from 1-grain simulation.

radial offset distance (r, actually equal to x) from the axis of the wake. In addition, the lines on this plot, show for the corresponding cases the pure electric field force $F = -Q\nabla\phi$ that would be exerted on a charge of this magnitude by the potential gradient, at the position of the downstream charge, if the potential were just what exists with a single upstream charge present. The lines are thus the electric wake-field-force ignoring the perturbation by the downstream grain and any transverse ion flow drag it experiences. Longitudinal ($\hat{\mathbf{z}}$) force perturbation is small except for the downstream particle when it is on axis in a strong density depression.

At $z = 2\lambda_{De}$ the wake grain is adjacent to the primary potential peak of the leading grain's wake. It is consequently *attracted* (since its charge is negative) in the transverse direction towards the axis of the wake, the potential peak. It experiences a negative force at positive r and a stable equilibrium at $r = 0$ (supposing that other forces constrain the grain's z-position to remain fixed.)

By contrast at $z = 3.5\lambda_{De}$ and $z = 5\lambda_{De}$ the wake grain is adjacent to a potential valley, and is *repelled* from the wake axis. The equilibrium at $r = 0$ is then unstable and the grain would be expected to find a stable equilibrium position where the force again crosses zero. This occurs near $r = 2\lambda_{De}$, although the simulation is not accurate enough to detect that zero-crossing unambiguously. Such observations appear to be capable of explaining the oblique alignment of grains at sufficient vertical separation[4, 11].

The total force obtained with two grain simulations (points) is slightly larger than the one-grain wake-field-force (lines) for $z/\lambda_{De} = 3.5$, by of order 20% at the peak. But differences are within uncertainties otherwise. These results thus contradict the factor-of-ten discrepancies reported in [8][15], even though the present simulations are at comparable parameters.

For smaller values of the nonlinearity parameter $\phi_p r_p/\lambda_{De}$, the discrepancy between total force and wake-field-force is proportionally even smaller. But because the total force decreases rapidly, approximately $\propto Q^2$, in the code it is overwhelmed by numerical noise before the fully linear regime is reached, preventing computational demonstration of the transition to linearity so far.

In summary, flow-perturbation causes at most a small fraction of the total transverse force on the wake particle, even in regimes with quite strong nonlinear saturation of the wake field. Therefore when grains are substantially smaller than the Debye length, it is still a reasonable approximation to take the transverse force to be given by the wake potential gradient alone, ignoring drag perturbations. The *nonlinear* single-grain potential must nevertheless be used.

[*] ihutch@mit.edu; http://www.psfc.mit.edu/~hutch/
[1] J. H. Chu and I. Lin, Phys. Rev. Lett. **72**, 4009 (1994).
[2] A. Melzer, V. A. Schweigert, I. V. Schweigert, A. Homann, S. Peters, and A. Piel, Phys. Rev. E **54**, R46 (1996).
[3] A. Melzer, V. A. Schweigert, and A. Piel, Phys. Rev. Lett. **83**, 3194 (1999).
[4] G. A. Hebner and M. E. Riley, Phys. Rev. E **68**, 046401 (2003).
[5] V. Steinberg, R. Sütterlin, A. V. Ivlev, and G. Morfill, Phys. Rev. Lett. **86**, 4540 (2001).
[6] A. A. Samarian, S. V. Vladimirov, and B. W. James, Phys. Plasmas **12**, 022103 (2005).
[7] M. Lampe, G. Joyce, and G. Ganguli, IEEE Trans. Plasma Sci. **33**, 57 (2005).
[8] G. Lapenta, Physical Review E **66**, 026409 (2002).
[9] S. A. Khrapak, A. V. Ivlev, S. K. Zhdanov, and G. E. Morfill, Phys. Plasmas, 042308(2005).
[10] I. H. Hutchinson, Plasma Phys. Control. Fusion **48**, 185 (2006).
[11] I. H. Hutchinson, Phys. Plasmas **18**, 032111 (2011).
[12] V. R. Ikkurthi, K. Matyash, A. Melzer, and R. Schneider, Phys. Plasmas, 103712(2010).
[13] W. J. Miloch, M. Kroll, and D. Block, Phys Plasmas, 103703(2010).
[14] R. W. Hockney and J. W. Eastwood, *Computer Simulation using Particles* (Taylor and Francis, London, 1988).
[15] Forces in Ref [8] were not derived from direct measurement in the code. They were obtained by applying analytic formulas using the code's flow.